Screened Collision-Induced Quantum Interference in Collisional Plasmas

Sang-Chul Na[a] and Young-Dae Jung[a,b]

[a] Department of Applied Physics, Hanyang University, Ansan, Kyunggi-Do 426-791, South Korea
[b] Atomic and Molecular Data Research Center, National Institute for Fusion Science, Toki, Gifu, 509-5292, Japan

Reprint requests to Y.-D. J.; E-mail: ydjung@hanyang.ac.kr

Z. Naturforsch. **64a,** 233 – 236 (2009); received April 8, 2008 / revised November 11, 2008

The effects of neutral particle collisions on the quantum interference in electron-electron collisions are investigated in collisional plasmas. The effective potential model taking into account the electron-neutral particle collision effects is employed in order to obtain the electron-electron collision cross section including the total spin states of the collision system. It is found that the collision effects significantly enhance the cross section. In addition, the collision-induced quantum interference effects are found to be significant in the singlet spin state. It is shown that the quantum interference effects decrease with increasing the thermal energy of the plasma. It is also shown that the quantum interference effects increase with an increase of the collision energy.

Key words: Quantum Interference; Collisional Plasmas.

Electron collisions [1 – 3] have received much attention since this process is one of the major physical processes in plasmas. It has been known that electron-electron collisions make contributions to the collective effects on the conductivity and to the electron-ion bremsstrahlung emission spectrum in plasmas [4]. In weakly coupled thermal plasmas, the screened interaction potential has been characterized by the Yukawa-type Debye-Hückel model [5] obtained by linearization of the Poisson equation with the Maxwell-Boltzmann velocity distribution. In addition, it has been shown that the far-field interaction potential in collisional plasmas may fall off as $1/r^2$ [6, 7], where r is the distance between the collision particles, due to the influence of collisions with neutral particles. Therefore, electron-electron interactions in collisional plasmas would be different from those in collisionless plasmas due to the influence of collisions with neutral atoms. Thus, in the present paper we consider the effects of electron-neutral particle collisions and the quantum interference on electron-electron collisions in collisional plasmas. The effective potential model [8] including the additional terms due to electron-neutral particle collisions apart from the conventional Debye-Hückel screening part is applied to describe the screened electron-electron interactions in collisional plasmas. Moreover, the direct and exchange scattering amplitudes due to the total spin states of the collision system are considered to obtain the electron-electron collision cross section as a function of the collision angle, collision energy, collision frequency, Debye length, and thermal energy.

For collisions in a potential field $V(\mathbf{r})$, the differential elastic collision cross section $\mathrm{d}\sigma$ per unit solid angle $\mathrm{d}\Omega$ is represented by [9, 10]

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = |f(\mathbf{k},\mathbf{k}')|^2, \tag{1}$$

where $f(\mathbf{k},\mathbf{k}')$ is the scattering amplitude, $\mathbf{k}$ and $\mathbf{k}'$ are, respectively, the wave vectors of the incident and scattered waves. In the first-order Born approximation [3], the scattering amplitude is given by

$$f(\mathbf{k},\mathbf{k}') = -\frac{\mu}{2\pi\hbar^2}\int \mathrm{d}^3\mathbf{r}\exp[\mathrm{i}(\mathbf{k}-\mathbf{k}')\cdot\mathbf{r}]V(\mathbf{r}), \tag{2}$$

where μ is the reduced mass of the collision system. The Born method is known to be reliable to provide a sufficiently accurate qualitative description of collision cross sections for weak interaction potentials, i. e., $|V|d/\hbar\upsilon_0 \ll 1$ [3], where $|V|$ is the typical strength of the potential, d is the range of the interaction potential, and υ_0 is the collision velocity. The useful analytical form of the effective screened potential [8] of a moving test charge in a warm collisional plasma has been obtained by the Poisson equation and plasma dielectric function $D(\mathbf{k},\omega)$ $[= 1-\omega_\mathrm{p}^2(\omega(\omega+\mathrm{i}\nu)-k^2\upsilon_T^2)^{-1}]$, where ω_p $[= (4\pi ne^2/m)^{1/2}]$ is the plasma frequency,

n is the electron number density, m is the mass of the electron, v is the collision frequency due to electron-neutral particle collisions, and υ_T [$= (k_\mathrm{B}T/m)^{1/2}$] is the thermal velocity, k_B is the Boltzmann constant, and T is the plasma temperature. Using the effective potential model [8], we obtain the electron-electron interaction potential $V(r,z)$ for all ranges of r including the additional terms due to the influence of electron-neutral particle collisions in warm collisional plasmas, if the collision velocity $\boldsymbol{\upsilon}_0$ ($= \upsilon_0\hat{z}$) is comparable but smaller than the thermal velocity υ_T and $vz < \upsilon_T$, i. e., the collision frequencies are small:

$$V(r,z) = \frac{e^2}{r}\mathrm{e}^{-r/\lambda_\mathrm{D}} + \frac{e^2 z}{r}\frac{v\upsilon_0\lambda_\mathrm{D}}{\upsilon_T^2}\left[\left(\frac{1}{r}+\frac{1}{2\lambda_\mathrm{D}}+\frac{\lambda_\mathrm{D}}{r^2}\right)\mathrm{e}^{-r/\lambda_\mathrm{D}} - \frac{\lambda_\mathrm{D}}{r^2}\right], \tag{3}$$

where λ_D [$= \upsilon_T/\omega_\mathrm{p} = (k_\mathrm{B}T/4\pi ne^2)^{1/2}$] is the Debye length. This effective interaction potential encompasses the several additional terms due to electron-neutral particle collisions apart from the standard Debye-Hückel shielding potential $(e^2/r)\mathrm{e}^{-r/\lambda_\mathrm{D}}$ since the Poisson equation for the electrostatic potential ϕ [8] for a test charge q_t in a warm collisional plasma would be given by $\nabla^2\phi = (q_\mathrm{t}/\lambda_\mathrm{D}^2)(\mathrm{e}^{-r/\lambda_\mathrm{D}}/r)[1 + (v\upsilon_0 z/2\upsilon_T^2)] - 4\pi q_\mathrm{t}\delta(\mathbf{r})$, where $\delta(\mathbf{r})$ is the delta function. Here, the electron-electron collision itself is assumed to be unaffected by collisions with neutral particles. After some mathematical manipulations in the cylindrical coordinates $\mathbf{r} = \rho\hat{\boldsymbol{\rho}} + z\hat{\boldsymbol{z}}$ with $\hat{\boldsymbol{\rho}}\cdot\hat{\boldsymbol{z}} = 0$, the scattering amplitude $f(K)$ for the electron-electron collision in collisional plasmas is found to be

$$f(k) = -\frac{\mu}{2\pi\hbar^2}I(K), \tag{4}$$

where K [$\equiv |\mathbf{k}-\mathbf{k}'| = 2k\sin(\Theta/2)$] is the momentum transfer, $|\mathbf{k}| = |\mathbf{k}'| \equiv k$, Θ is the scattering angle between the incident and scattered directions measured in the center of the mass system, and the integral function $I(K)$ is represented by

$$\begin{aligned} I(K) &= 2\pi e^2\int_0^\infty \mathrm{d}\rho\,\rho\int_{-\infty}^\infty \mathrm{d}z\,\mathrm{e}^{\mathrm{i}Kz}\left\{\frac{1}{(\rho^2+z^2)^{1/2}}\mathrm{e}^{-(\rho^2+z^2)^{1/2}/\lambda_\mathrm{D}}\right. \\ &\quad\left.+\frac{v\upsilon_0\lambda_\mathrm{D}}{\upsilon_T^2}\left[\left(\frac{z}{\rho^2+z^2}+\frac{z}{2\lambda_\mathrm{D}(\rho^2+z^2)^{1/2}}+\frac{z\lambda_\mathrm{D}}{(\rho^2+z^2)^{3/2}}\right)\mathrm{e}^{-(\rho^2+z^2)^{1/2}/\lambda_\mathrm{D}} - \frac{z\lambda_\mathrm{D}}{(\rho^2+z^2)^{3/2}}\right]\right\} \\ &= 4\pi e^2\lambda_\mathrm{D}\int_0^\infty \mathrm{d}z\left[\mathrm{e}^{-z/\lambda_\mathrm{D}}\cos(Kz) - \mathrm{i}\frac{v\upsilon_0\lambda_\mathrm{D}}{\upsilon_T^2}\left(\frac{z}{2\lambda_\mathrm{D}}\mathrm{e}^{-z/\lambda_\mathrm{D}}+\mathrm{e}^{-z/\lambda_\mathrm{D}}-1\right)\sin(Kz)\right]. \end{aligned} \tag{5}$$

After some algebra, the scattering amplitude $f(\bar{K})$ is then obtained as follows:

$$f(\bar{K}) = -a_0\bar{\lambda}_\mathrm{D}^2\left\{\frac{1}{1+(\bar{K}\bar{\lambda}_\mathrm{D})^2} - \mathrm{i}\frac{\bar{v}\upsilon_0\bar{\lambda}_\mathrm{D}}{\upsilon_T}\frac{[1+4(\bar{K}\bar{\lambda}_\mathrm{D})^2+2(\bar{K}\bar{\lambda}_\mathrm{D})^4]}{(\bar{K}\bar{\lambda}_\mathrm{D})[1+(\bar{K}\bar{\lambda}_\mathrm{D})^2]^2}\right\}, \tag{6}$$

where $\bar{K} \equiv Ka_0$ is the scaled momentum transfer, a_0 ($= \hbar^2/me^2$) is the first Bohr radius of the hydrogen atom, $\bar{\lambda}_\mathrm{D}$ ($\equiv \lambda_\mathrm{D}/a_0$) is the scaled Debye length, and $\bar{v}$ ($\equiv va_0/\upsilon_T$) is the scaled collision frequency. For the identical particle collisions, the projectile particle would not be distinguished from the target particle due to the indistinguishability of identical quantum particles. Hence, in addition to the direct scattering amplitude $f_\mathrm{d}(\Theta)$, the exchange scattering amplitude $f_\mathrm{ex}(\pi-\Theta)$ has also to be considered in evaluating the total electron-electron collision cross section. Since the two-electron system would be in the spin-antisymmetric state (singlet state) and in the spin-symmetric state (triplet state), the differential elastic electron-electron collision cross section $\mathrm{d}\sigma$ in the centre-of-mass (CM) system is given by the contributions of the singlet and triplet spin states [9]:

$$\begin{aligned} \left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)_\mathrm{CM} &= \frac{1}{4}|f_\mathrm{d}(\Theta)+f_\mathrm{ex}(\pi-\Theta)|^2 + \frac{3}{4}|f_\mathrm{d}(\Theta)-f_\mathrm{ex}(\pi-\Theta)|^2 \\ &= |f_\mathrm{d}(\Theta)|^2 + |f_\mathrm{ex}(\pi-\Theta)|^2 - \mathrm{Re}[f_\mathrm{d}(\Theta)f_\mathrm{ex}^*(\pi-\Theta)], \end{aligned} \tag{7}$$

where the direct scattering amplitude $f_\mathrm{d}(\Theta)$ represents the incoming electron scattered into the direction Θ in the centre-of-mass system, the exchange scattering amplitude $f_{\mathrm{ex}}(\pi-\Theta)$ represents the scattering into the direction $\pi-\Theta$, Re represents the real part of the product of the scattering amplitudes, * stands for the complex conjugate, and $\mathrm{Re}[f_{\mathrm{d}}(\Theta) f_{\mathrm{ex}}^{*}(\pi-\Theta)]$ represents the quantum interference effects due to the total spin states of the collision system. From (7), it can be understood that the scattered electrons for the scattering angle $\Theta=\pi/2$ would be in the singlet spin state since the triplet spin state vanishes at $\Theta=\pi/2$. It has been shown that the relation between the differential elastic cross section for the identical particle collisions in the laboratory system (L) and that in the centre-of-mass system is represented by the expression [11]

$$\left[\frac{\mathrm{d}\sigma(\theta_{\mathrm{L}})}{\mathrm{d}\Omega}\right]_{\mathrm{L}} = 4\cos\theta_{\mathrm{L}}\left[\frac{\mathrm{d}\sigma(\Theta)}{\mathrm{d}\Omega}\right]_{\mathrm{CM}}, \tag{8}$$

where θ_{L} $(=\Theta/2)$ is the scattering angle in the laboratory system. After some algebra, the differential cross section $(\mathrm{d}\sigma/\mathrm{d}\Omega)_{\mathrm{L}}$ in units of πa_0^2 for the elastic electron-electron collisions in the laboratory system including the effects of the electron-neutral particle collisions and quantum interference in collisional plasmas is found to be

$$\begin{aligned}
\left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)_{\mathrm{L}}/\pi a_0^2 = \frac{\cos\theta_{\mathrm{L}}}{\pi}\bar{\lambda}_{\mathrm{D}}^4 \Bigg\{ &\left[\frac{1}{(1+2\bar{E}\bar{\lambda}_{\mathrm{D}}^2\sin^2\theta_{\mathrm{L}})^2} + \frac{4\bar{\nu}^2\bar{E}\bar{\lambda}_{\mathrm{D}}^2}{\bar{E}_T}\frac{(1+2\bar{E}\bar{\lambda}_{\mathrm{D}}^2\sin^2\theta_{\mathrm{L}}+\bar{E}^2\bar{\lambda}_{\mathrm{D}}^4\sin^4\theta_{\mathrm{L}}/2)^2}{(\sqrt{2}\bar{E}^{1/2}\bar{\lambda}_{\mathrm{D}}\sin\theta_{\mathrm{L}})^2(1+2\bar{E}\bar{\lambda}_{\mathrm{D}}^2\sin^2\theta_{\mathrm{L}})^4}\right] \\
&+\left[\frac{1}{(1+2\bar{E}\bar{\lambda}_{\mathrm{D}}^2\cos^2\theta_{\mathrm{L}})^2} + \frac{4\bar{\nu}^2\bar{E}\bar{\lambda}_{\mathrm{D}}^2}{\bar{E}_T}\frac{(1+2\bar{E}\bar{\lambda}_{\mathrm{D}}^2\cos^2\theta_{\mathrm{L}}+\bar{E}^2\bar{\lambda}_{\mathrm{D}}^4\cos^4\theta_{\mathrm{L}}/2)^2}{(\sqrt{2}\bar{E}^{1/2}\bar{\lambda}_{\mathrm{D}}\cos\theta_{\mathrm{L}})^2(1+2\bar{E}\bar{\lambda}_{\mathrm{D}}^2\cos^2\theta_{\mathrm{L}})^4}\right] \\
&-\Bigg[\frac{1}{(1+2\bar{E}\bar{\lambda}_{\mathrm{D}}^2\sin^2\theta_{\mathrm{L}})}\frac{1}{(1+2\bar{E}\bar{\lambda}_{\mathrm{D}}^2\cos^2\theta_{\mathrm{L}})} \\
&\quad -\frac{4\bar{\nu}^2\bar{E}\bar{\lambda}_{\mathrm{D}}^2}{\bar{E}_T}\cdot\frac{1+2\bar{E}\bar{\lambda}_{\mathrm{D}}^2\sin^2\theta_{\mathrm{L}}+\bar{E}^2\bar{\lambda}_{\mathrm{D}}^4\sin^4\theta_{\mathrm{L}}/2}{\sqrt{2}\bar{E}^{1/2}\bar{\lambda}_{\mathrm{D}}\sin\theta_{\mathrm{L}}(1+2\bar{E}\bar{\lambda}_{\mathrm{D}}^2\sin^2\theta_{\mathrm{L}})^2}\cdot\frac{1+2\bar{E}\bar{\lambda}_{\mathrm{D}}^2\cos^2\theta_{\mathrm{L}}+\bar{E}^2\bar{\lambda}_{\mathrm{D}}^4\cos^4\theta_{\mathrm{L}}/2}{\sqrt{2}\bar{E}^{1/2}\bar{\lambda}_{\mathrm{D}}\cos\theta_{\mathrm{L}}(1+2\bar{E}\bar{\lambda}_{\mathrm{D}}^2\cos^2\theta_{\mathrm{L}})^2}\Bigg]\Bigg\},
\end{aligned} \tag{9}$$

where $\bar{E}\equiv E/Ry$, E $(=\mu v_0^2/2)$ is the collision energy, Ry $(=me^4/2\hbar^2\approx 13.6$ eV) is the Rydberg constant, $\bar{E}_T\equiv E_T/Z^2Ry$, and E_T $(=k_{\mathrm{B}}T)$ is the thermal energy.

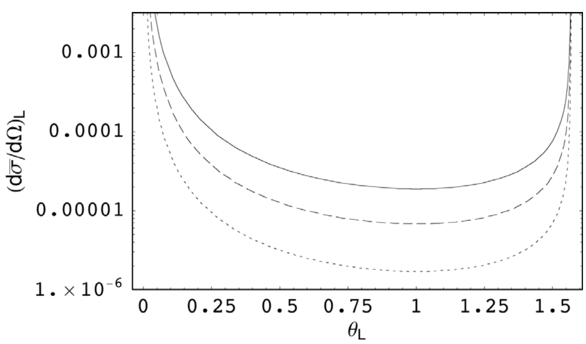

Fig. 1. The scaled differential elastic electron-electron collision cross section $(\mathrm{d}\bar{\sigma}/\mathrm{d}\Omega)_{\mathrm{L}}$ in units of πa_0^2 in the laboratory system as a function of the scattering angle θ_{L} in units of radians for $E=136$ eV, $T=3.2\cdot 10^6$ K, and $n=5.3\cdot 10^{26}$ m^{-3}. The dotted line represents the case of $\bar{\nu}=3\cdot 10^{-2}$. The dashed line represents the case of $\bar{\nu}=6\cdot 10^{-2}$. The solid line represents the case of $\bar{\nu}=10^{-1}$.

Figure 1 represents the scaled differential elastic electron-electron collision cross section $(\mathrm{d}\bar{\sigma}/\mathrm{d}\Omega)_{\mathrm{L}}$ $[\equiv (\mathrm{d}\sigma/\mathrm{d}\Omega)_{\mathrm{L}}/\pi a_0^2]$ in units of πa_0^2 in the laboratory system including the effects of neutral particle collisions

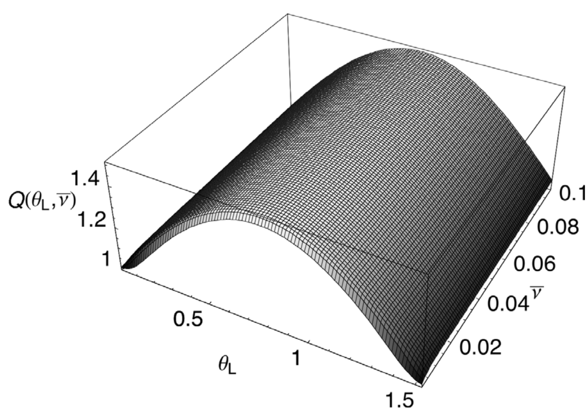

Fig. 2. The surface plot of the quantum interference effect $Q(\theta_{\mathrm{L}},\bar{\nu})$ as a function of the scattering angle θ_{L} in units of radians and the scaled collision frequency $\bar{\nu}$ for $E=136$ eV, $T=3.2\cdot 10^6$ K, and $n=5.3\cdot 10^{26}$ m^{-3}.

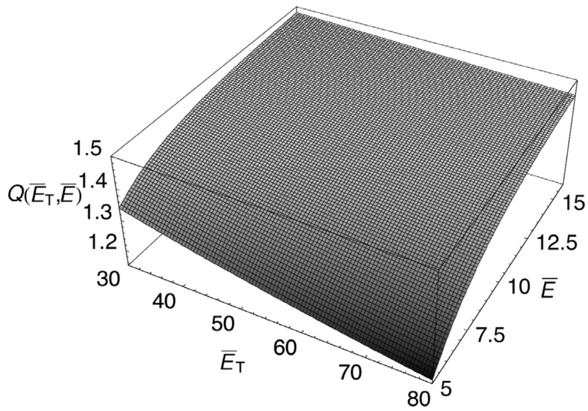

Fig. 3. The surface plot of the quantum interference effect $Q(\bar{E}_T,\bar{E})$ as a function of the scaled thermal energy $\bar{E}_T$ and the scaled collision energy $\bar{E}$ for $\theta_L = \pi/4$, $\bar{v} = 10^{-3}$, $n = 5.3 \cdot 10^{26}$ m^{-3}, and $T = 3.2 \cdot 10^6$ K.

and quantum interference in collisional plasmas as a function of the scattering angle θ_L in units of radians. As it is seen, the collision effects due to neutral particles significantly enhance the collision cross section. Figure 2 shows the surface plot of the quantum interference effect $Q(\theta_L,\bar{v})$ [$\equiv (d\sigma/d\Omega)_L/(d\sigma'/d\Omega)_L$] as a function of the scattering angle θ_L and the scaled collision frequency $\bar{v}$, where $(d\sigma'/d\Omega)_L$ is the differential elastic electron-electron collision cross section without the effect of the quantum indistinguishability. From this figure, it is found that the quantum interference effects are quite significant near the scattering angle $\theta_L = \pi/4$, i. e., in the singlet spin state of the collision system. In addition, Fig. 3 represents the surface plot of the quantum interference effect $Q(\bar{E}_T,\bar{E})$ as a function of the scaled thermal energy $\bar{E}_T$ and the scaled collision energy $\bar{E}$. As shown, it is found that the quantum interference effects on the cross section decrease with increasing thermal energy. It is also shown that the quantum interference effects increase with an increase of the collision energy.

Hence, we have found that the effects of neutral particle collisions and quantum interference play important roles in electron-electron collisions in plasmas. Therefore, the quantum interference effects should be appropriately included in the estimations of the correct collision cross sections and reaction rates in plasmas. These results provide useful information on the effects of neutral particle collisions and quantum interference on the collisions of quantum indistinguishable particles in collisional plasmas.

Acknowledgements

Y.-D. Jung gratefully acknowledges Director-General Professor O. Motojima, Director Professor M. Sato, Director Professor Y. Hirooka, Professor I. Murakami, and Professor D. Kato for warm hospitality and support while visiting the National Institute for Fusion Science (NIFS), Japan, as a long-term visiting professor. This work was supported by the research fund of Hanyang University (HYU-2008-T).

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